GCSE Maths – Algebra

Gradients and Areas of Graphs in Context

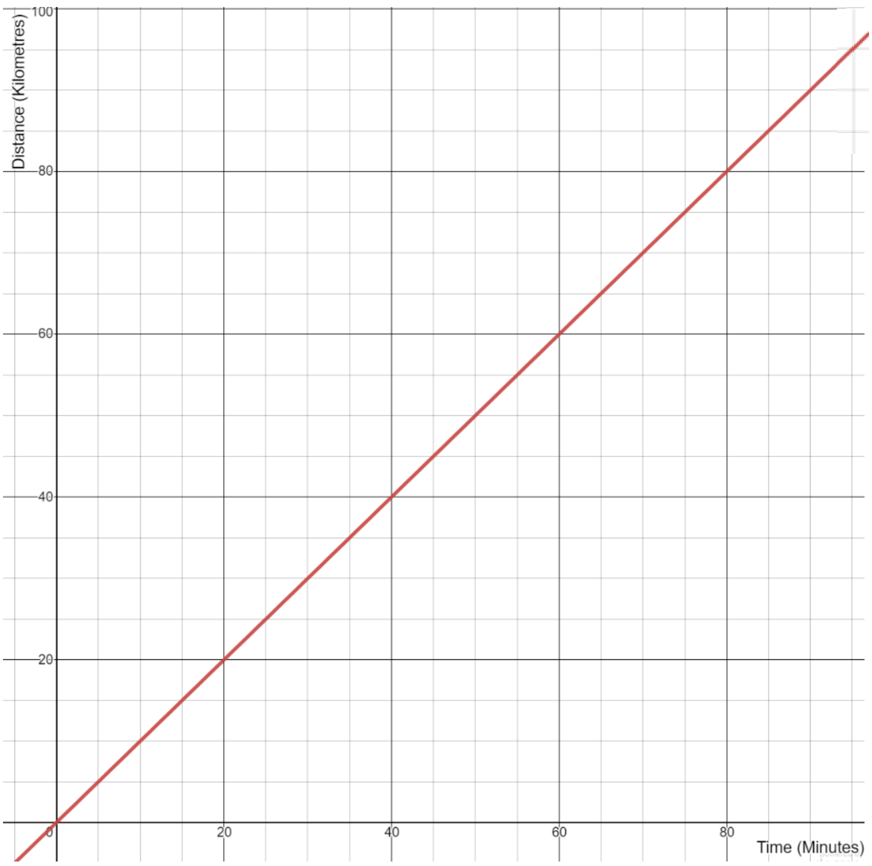
(Higher Only)

Notes

WORKSHEET

Real-life Graphs

Graphs can be used to show the **relationships** in real life between two variables. For example, consider the following distance-time graph for a car.



The x-axis represents **time** (in minutes), and the y-axis is **distance** (in kilometres). By looking at a particular point on the line, we can calculate how far the car travels in a certain amount of time. Here, if we look at the point (40, 40), we can work out that the car has travelled 40 km in 40 minutes.

The **gradient** of this line tells us about the **change in distance in a period of time**, which is the car's **speed**. To calculate this, work out the gradient by calculating the **difference in time** (y) between two points divided by the **difference in distance** (x) between the **same two points**. For this, we'll use (0, 0) and (20, 20).

$$\frac{Change\ in\ y}{Change\ in\ x} = \frac{20-0}{20-0} = \frac{20}{20} = \mathbf{1\ km/min}$$

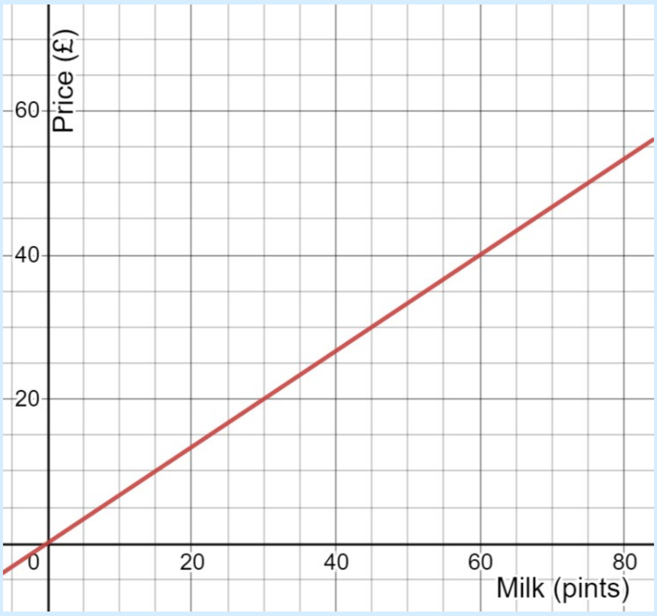
This could be converted to a more standard unit:

$$1\ \text{km/min} = \ 60\ \text{km/h}$$

Example: Using the graph below, calculate the following:

a) the gradient, and explain what it means in context

b) the price of 100 pints of milk



a) To calculate the gradient, we need to identify two points on the line, then calculate difference in y divided by difference in x.

We will us the points $(0, 0)$ *and* $(60, 40)$*:*

$$\frac{Change\ in\ y}{Change\ in\ x} = \frac{40 - 0}{60 - 0} = \frac{40}{60} = \frac{2}{3}$$

The gradient is $\frac{2}{3}$*.*
This tells us about the change in price (in pounds) divided by the pints of milk, which is the *<u>price per pint</u>* *(£/pint).*

b) To find the price of 100 pints, we can find the equation of the line and substitute in $x = 100$.

The basic equation of a line is $y = mx + c$*.*

As the line passes through (0, 0) there is no y-intercept so $\boldsymbol{c = 0}$*.*
From a) the gradient was found to be $\boldsymbol{m} = \frac{2}{3}$*.*
Therefore, the equation of this line is

$$\boldsymbol{y = \frac{2}{3}x}$$

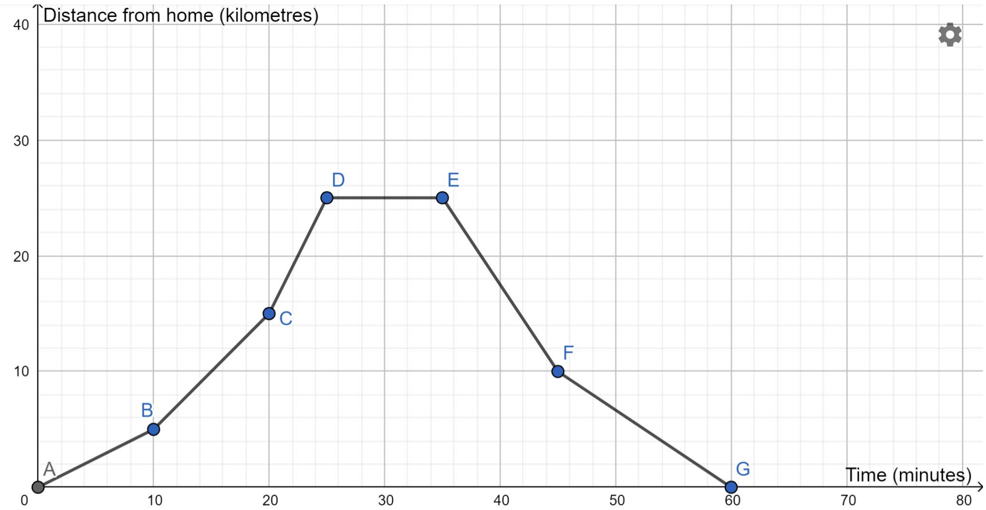
Now, substitute in $x = 100$ *into the equation of the line and find the price.*

$$y = \frac{2}{3} \times 100 = \pounds\mathbf{66.67}$$

Displacement-time graphs

A displacement-time graph is a type of graph that shows **how far** an object is from its **starting place** (e.g. home). As the object returns to the starting position, the line of the graph will go **down** towards the x-axis.

The following graph shows the displacement-time graph of a car.



We can work out the **speed** of each part of the journey by calculating the **gradient** of each separate section. For example, let's work out the speed between points C and D.

$$Point\ C = (20, 15)$$
$$Point\ D = (25, 25)$$

$$\frac{Change\ in\ y}{Change\ in\ x} = \frac{10}{5} = 2\ \text{km/min}$$

Between points D and E, the car is stationary at 25 km from home for 10 minutes.

When the car is travelling home, the **gradient of the line is negative**, as seen between points E and G. Let's work out the speed between points F and G.

$$Point\ F = (45, 10)$$
$$Point\ G = (60, 0)$$

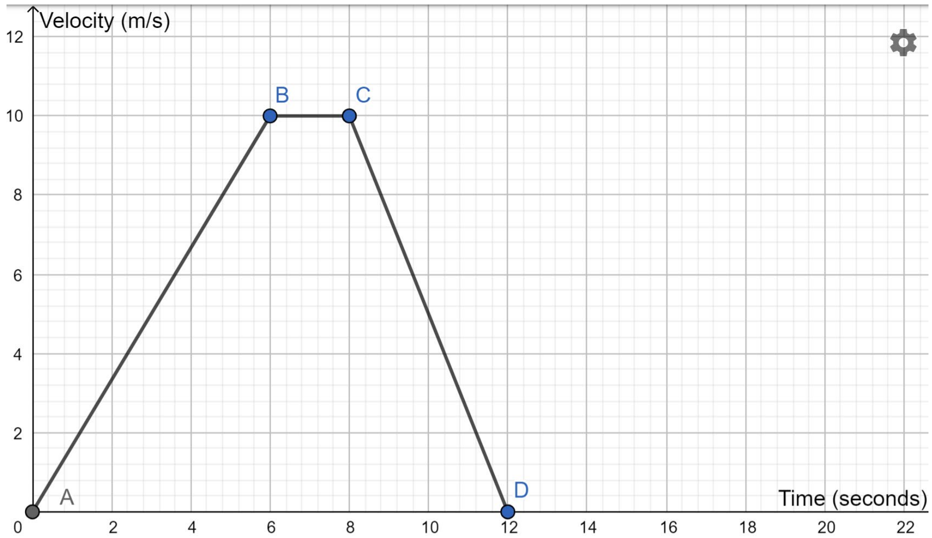
$$\frac{Change\ in\ y}{Change\ in\ x} = \frac{-10}{15} = -\frac{2}{3}$$

$Speed = \frac{2}{3}\ \text{km/min}$ (as speed cannot be negative)

However, if we are asked to find the **velocity** of the car when it travels back to home, it can be a **negative** value. This is because velocity is the **speed in a given direction**. If that given direction is away from home, then travelling **back to home** will give a negative velocity.

Velocity-time graphs

A velocity time graph has velocity on the y-axis and time on the x-axis. For example, look at the following graph:



- Between points A and B, the object is **accelerating**, as its **velocity is increasing** over time.
- Between points B and C, the object is travelling as a constant velocity for 2 seconds.
- Between points C and D, the object is **decelerating**, as its **velocity is decreasing** over time.

The lines between points A and B, and C and D, are straight. This means the **acceleration** is **constant**. To find the acceleration, we divide the change in velocity by the time.

$$Acceleration\ between\ A\ and\ B = \frac{Change\ in\ y}{Change\ in\ x} = \frac{10}{6} = 1.67\ \mathrm{m/s^2}$$

$$Acceleration\ between\ C\ and\ D = \frac{Change\ in\ y}{Change\ in\ x} = \frac{-10}{4} = -2.5\ \mathrm{m/s^2}$$

A **negative acceleration** is a **deceleration**. Don't forget to use the correct **units** for acceleration or deceleration. The units are dependent on the measure of distance and time used but have the general form $distance/time^2$, e.g. m/s^2, km/h^2.

From velocity-time graphs, we can calculate the **total distance** travelled by calculating the **area under the curve**. We can section each part of the graph into triangles and rectangles, and then add the total area of each shape together.

$$Distance\ travelled\ between\ A\ and\ B = \frac{1}{2} \times 6 \times 10 = 30\ \mathrm{metres}$$
$$Distance\ travelled\ between\ B\ and\ C = 2 \times 10 = 20\ \mathrm{metres}$$
$$Distance\ travelled\ between\ C\ and\ D = \frac{1}{2} \times 4 \times 10 = 20\ \mathrm{metres}$$
$$\boldsymbol{Total\ distance = 30 + 20 + 20 = 70\ \mathbf{metres}}$$

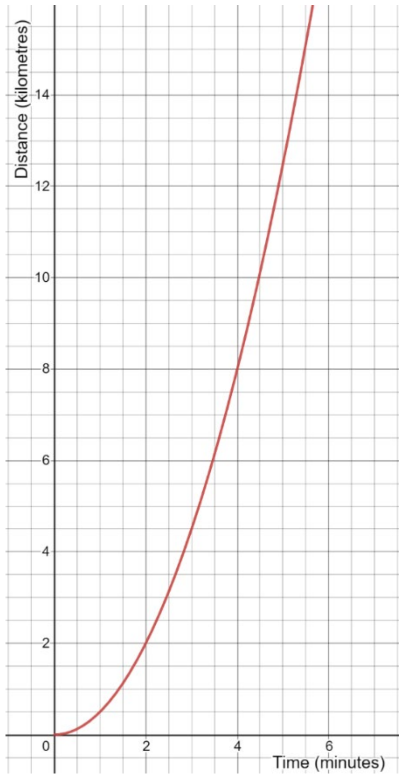
Non-linear graphs

Not all graphs will have straight lines that we can easily work out the gradient from. For example, consider the following distance-time graph of a car.

The car is increasing in speed as time goes on, which we can see by looking at the **changing gradient** of the line.

To work out the speed at a given time, we need to draw a **tangent** to the line.

A tangent is a **straight line** that just **touches** the curve at a particular point. It has the **same gradient** as the point on the curve that it touches.



Let's work out the **speed the car is travelling at the 4th minute**. Use a ruler to draw a tangent to the line at the point (4, 8).

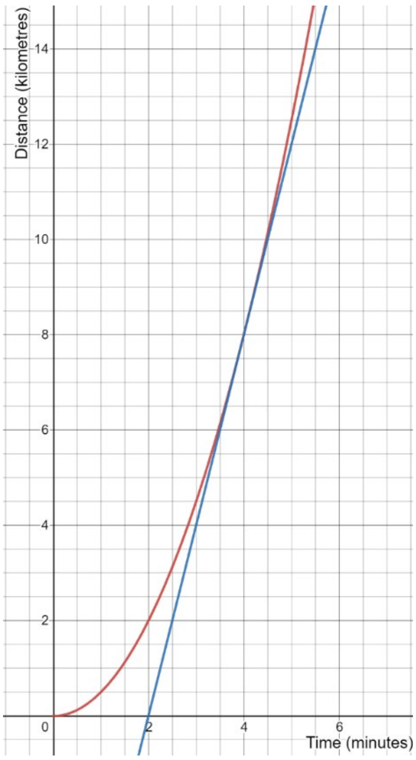
Now we are going to find the **gradient of the tangent** to the point (4,8).

Identify another point that the tangent line passes through, such as (2, 0). Using this coordinate and the point (4, 8), we can calculate the gradient of the line at (4,8):

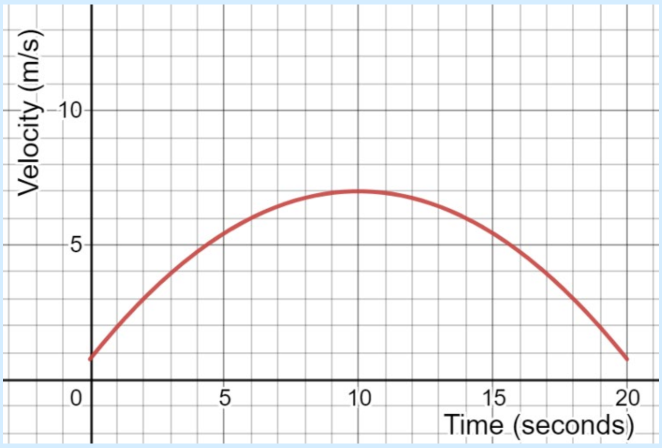
$$\frac{Change\ in\ y}{Change\ x} = \frac{8-0}{4-2} = \frac{8}{2} = 4$$

The **gradient** at the point (4, 8) is equal to 4.

This means that **speed** of the car at the 4th minute is $\mathbf{4\ km/min.}$



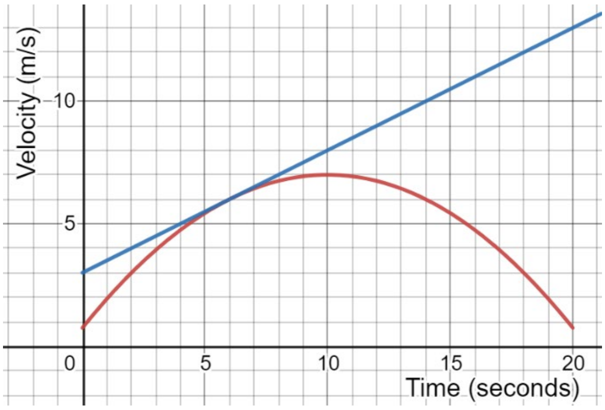
Example: A velocity-time graph of an object is shown below.
Find the acceleration of the object at 6 seconds.



1. Identify the point on the line which shows the velocity at 6 seconds.

   The point is (6, 6).

2. Draw a tangent at the point found.



3. Identify other points that lie on this line so that we can calculate the gradient of this tangent.

   Let's use the points (6, 6) and (10, 8):

$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{8-6}{10-6} = \frac{2}{4} = \frac{1}{2}$$

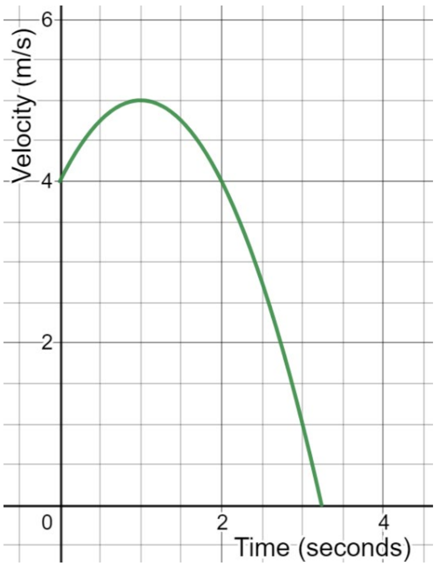
The gradient at the point (6, 6) is $\frac{1}{2}$. This is the acceleration, because acceleration is the change in velocity (change in y) divided by the change in time (change in x), or the gradient.

Don't forget to use the correct units!

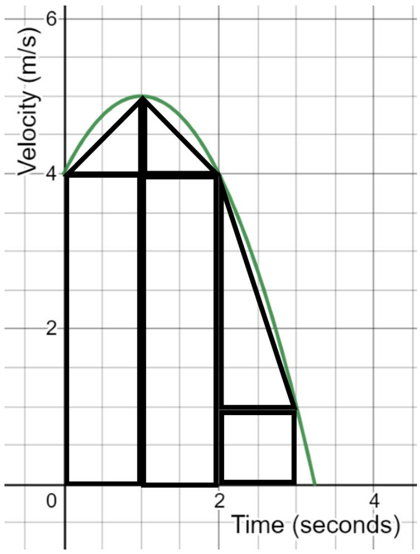
$$\boldsymbol{Acceleration = 0.5\ \mathrm{m/s^2}}$$

You may also be asked to find the **area** under the curve in a **non-linear** graph. In this case, we cannot find the area exactly, but instead will **estimate** it.

Consider the velocity-time graph below.



To find the total distance travelled by this object, we find the area under the curve. We **fit shapes under the curve** to make an **estimation** of the area, like this:



Work out the area of each shape individually, then add them together:

$$\boldsymbol{Rectangles}: (4 \times 1)\ \mathrm{m}^2 \times 2 = 4\ \mathrm{m}^2 \times 2 = 8\ \mathrm{m}^2$$

$$\boldsymbol{Top\ triangles}: \left(\frac{1}{2} \times 1 \times 1\right) \mathrm{m}^2 \times 2 = \frac{1}{2}\ \mathrm{m}^2 \times 2 = 1\ \mathrm{m}^2$$

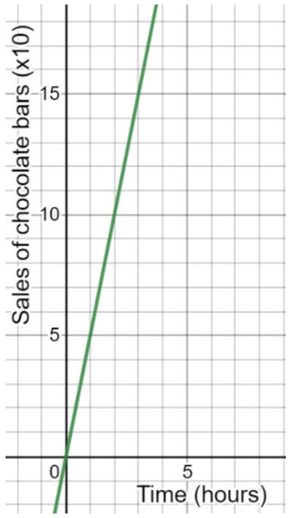
$$\boldsymbol{Right\ side\ triangle}: \frac{1}{2} \times 1 \times 3 = 1.5\ \mathrm{m}^2$$

$$\boldsymbol{Square}:\ 1 \times 1 = 1\ \mathrm{m}^2$$

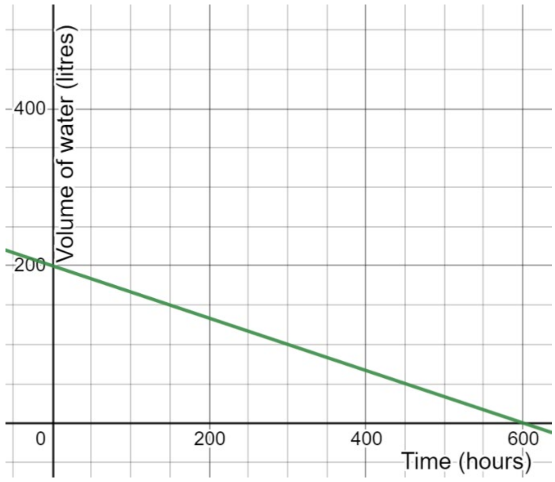
$$\boldsymbol{Total\ Area\ (estimation)}:\ 8\ \mathrm{m}^2 + 1\ \mathrm{m}^2 + 1.5\ \mathrm{m}^2 + 1\ \mathrm{m}^2 = \mathbf{11.5\ m^2}$$

Gradients and Areas in Context – Practice Questions

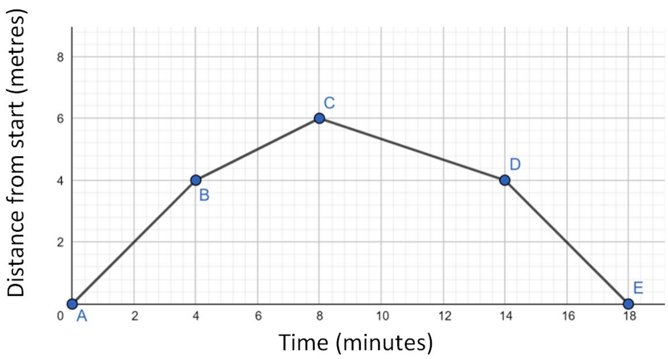
1. Calculate the rate of sales of chocolate bars.



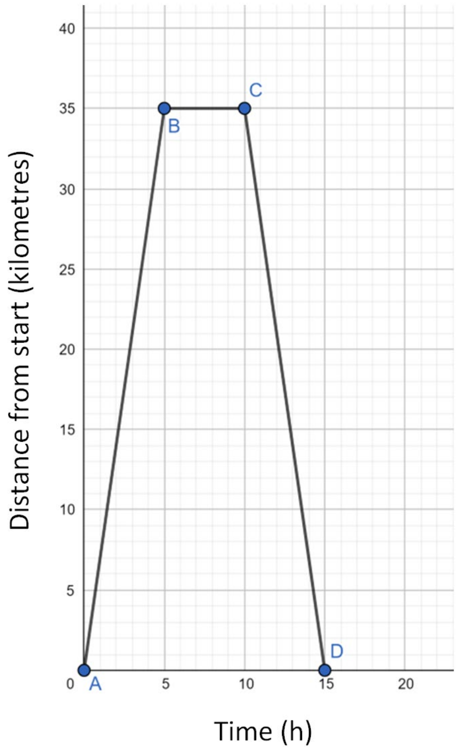
2. Calculate the rate of water being emptied from a tank



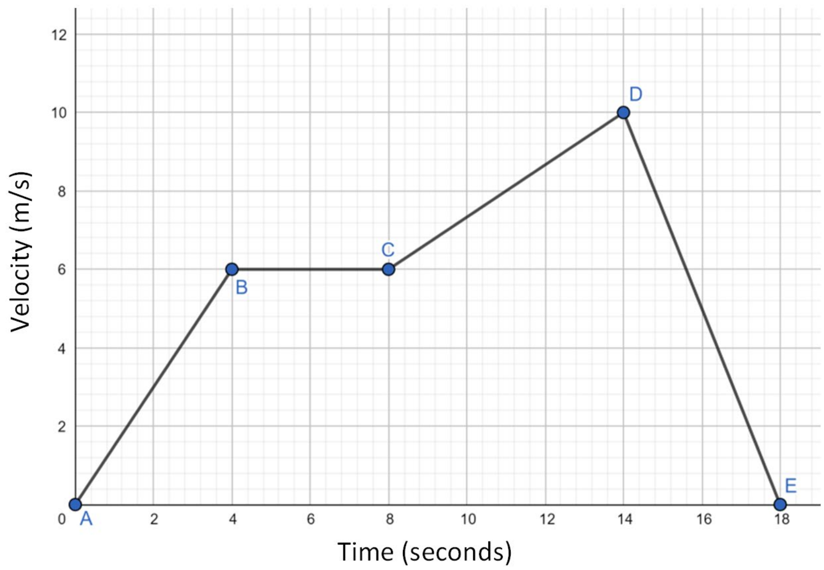
3. Calculate the speed between the points C and D.



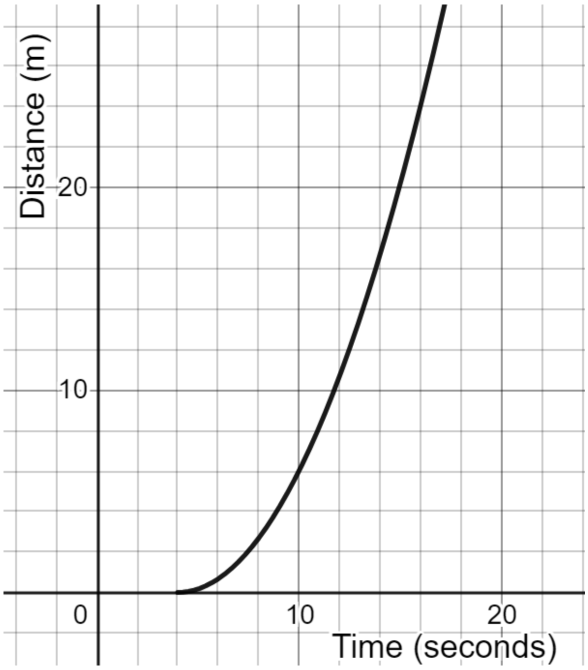
4. Calculate the speed of this car between points A and B. What is happening between points B and C?



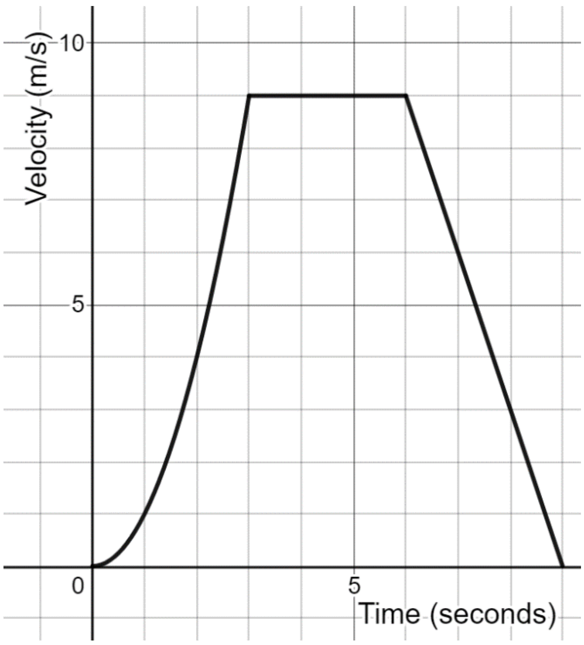
5. Find the deceleration between points D and E, and calculate the total distance travelled by this object.



6. Below is a distance-time graph of an object. Calculate the speed at the 14th second.



7. Find the total distance travelled by the object in this graph:



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.